Optimized Prediction for Geometry Compression of Triangle Meshes

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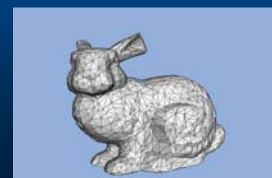
Graphics Compression for 3D Triangle Meshes

- Graphics compression is an emerging need for storing, transmitting, and visualizing large graphics models.
- 3D triangle mesh:
  - The most common type of graphics models
  - Two components of information:
    geometry -- 3D coordinates of mesh vertices
    connectivity – edges & triangles connecting vertices



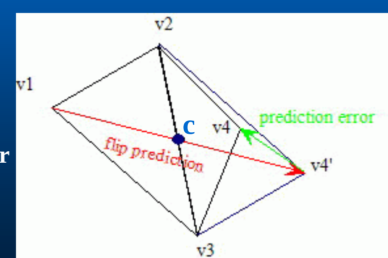
Previous Work

- Lots of results in connectivity compression… (see paper)
- Best connectivity compression results: 1.5 – 4 bits per vertex on an average
  - e.g. [Taubin-Rossignac 98], [Touma-Gotsman 98], [Rossignac 99], [Alliez-Desbrun 01]
- Geometry compression results are not equally impressive
  - Usually quantize each coordinate to a 10-bit or 12-bit integer (30 or 36 bits/vertex in raw data)
  - Typical results: 40—50% of raw data (12—18 bits/vertex)
    e.g. [Deering 95], [Karni-Gotsman 00], [Taubin-Rossignac 98], [Touma-Gotsman 98]

Geometry compression is by far the dominating bottleneck!!

Previous Work: Geometry Compression (1)

- Flipping method [Touma-Gotsman 98]
  - Dominant, widely considered state of the art; adopted to the MPEG-4 standard for mesh geometry coding
  - Traverse triangles by connectivity coder; predict new vertex position of new triangle by flipping using parallelogram rule
  - Drawback: triangle traversal ignores the geometry of the model
- Other extensions of flipping
  - [Isenburg-Alliez 02]: beyond triangle meshes
  - [Isenburge-Gumhold 03]: out-of-core method for meshes larger than main memory
  - * Do not address the drawback



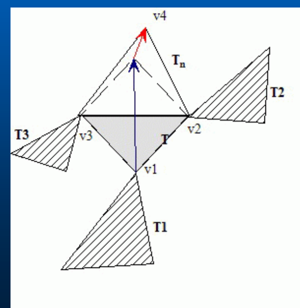
Previous Work: Geometry Compression (2)

- Prediction tree method [Kronrod-Gotsman 02]
  - Only previous work trying to optimize the flipping prediction error
  - Formulate the problem as finding an optimal cover tree
  - Take the dual graph of the triangle mesh, span the mesh triangles (nodes in dual graph) until all vertices are covered, with min total dual-edge cost (prediction error)
  - Heuristic solution; improves the flipping approach

- Sub-optimal:

• May cover vertices more than once

• Cannot visit a triangle from a vertex-adjacent neighbor



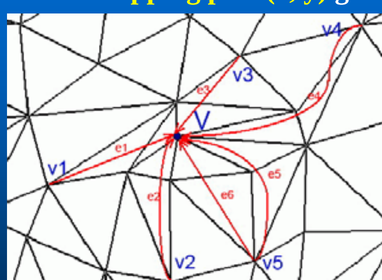
Our New Algorithm

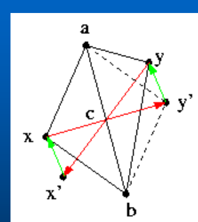
- Try to optimize the flipping prediction error
  - New formulation: finding a constrained minimum spanning tree on a new graph G (G is not the dual graph)
  - Span each vertex exactly once (vs. cover more than once)
  - Can visit a triangle from vertex-adjacent neighbor (vs. cannot)
  - Improves the prediction tree method by up to 33.2%

- Overview: 3 major technical components
  - Problem formulation: finding a constrained minimum spanning tree (CMST) on the graph G
  - Heuristic algorithm to find an approximate CMST on G
  - Algorithm to traverse CMST in another pass, build a pseudo-CMST & collect left-over triangles in the same pass, and finish both geometry and connectivity coding

Problem Formulation

- Observation: many possible ways of flipping for a vertex
  - Each flipping pair (x, y) gives a possible flipping





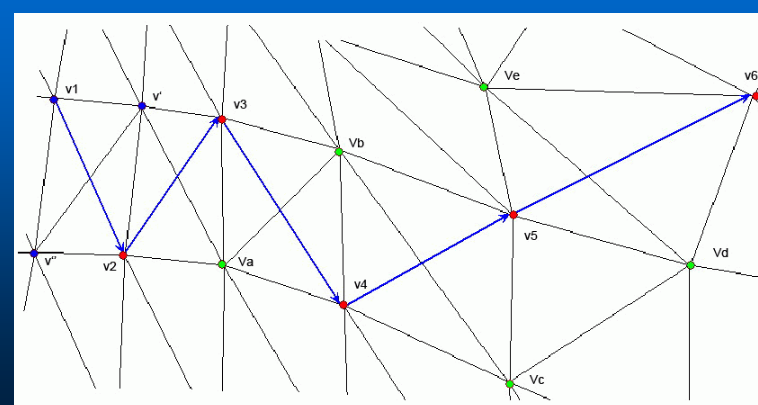
• Form a graph G:

* nodes---mesh vertices; edges---connect all flipping pairs, edge cost = prediction error

* (y', y) = (x', x) ➔ G is undirected * minimum spanning tree on G

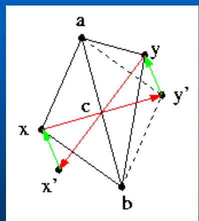
Problem Formulation (cont.)

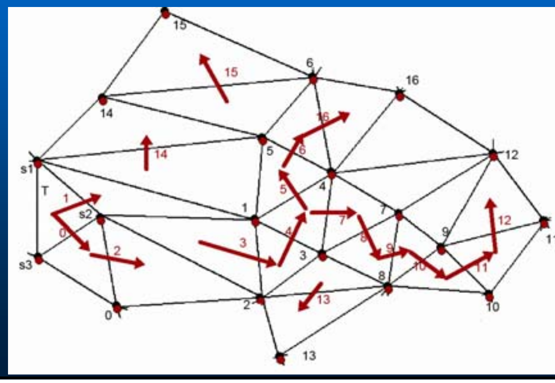
Not correct yet… the flipping constraint!



Final Problem Formulation

- In graph G, each edge (x, y) has **constraint vertices a, b**
- **Constrained minimum spanning tree T** on G: T **admits** a traversal where **each (x, y) is visited only after visiting a, b**





an example of CMST T

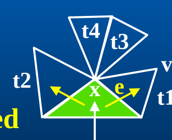
Heuristic Algorithm for CMST

- **Modify Prim's algorithm** for an **approx. CMST T**
  - For each edge (x, y) of G, make **bidirectional links** between **(x, y)** and its **constraint vertices a, b**
  - Initially, include 3 vertices of a triangle to T for initial prediction
  - Use a **priority queue Q** to maintain vertices not yet added to T
  - **Key (x)**: min cost of adding x to T, initially **infinity**; key (x) ← min { cost (x, y) | (x, y) is **valid**, i.e., **y, a, b already in T**}
  - While Q is not empty do
    - » v ← Extract-min (Q); include v to T
    - » Update key values of vertices **influenced** by v (**candidates** for **newly valid** edges: **(i) edges incident on v; (ii) edges with v a constraint vertex**)
    - If key (v) = infinity, then start a new tree (rarely occurred)
- Cost (T) is very close to the cost of **unconstrained** MST (**unachievable lower bound**)

Pseudo-CMST and Final Encoding

- The approx. CMST T admits a **valid traversal** by the order **we grow T**
- This order grows the **boundary** of the patch of current T **arbitrarily---very expensive to encode**

- Idea: each triangle has **at most 3 edges** to flip
- Traverse T in another pass; build a **pseudo CMST Tp** & collect **left-over** triangles
- **(i) recursively traverse t1; (ii) recursively traverse t2; (iii) collect t3, t4 if all vertices visited**



* Step (i): if t1 is visited, ignore t1; else

- If v unvisited: **(a) e in T:** predict v by e, add v, e to Tp, recurse from t1 **(b) e not in T:** ignore (v, t1 will be visited later by other paths)
- If v visited: add e to Tp with **no cost (pseudo-edge), recurse from t1**

Summary: Algorithm Steps

(1) Form graph G

(2) Compute an approximate CMST

(3) Compute a pseudo-CMST & collect left-over triangles, finish geometry & connectivity coding

Experiments

- 12 datasets commonly used in literature
  - size: small --- moderately large
  - feature: smooth --- with significantly many sharp corners
- Vertex coordinates are quantized to 12-bit integers
- Compare first-order entropy of prediction errors of:
  - constrained MST (CMST) vs.
    unconstrained MST (lower bound, though unachievable)
  - pseudo-CMST vs.
    flipping [Touma-Gotsman 98] (code available from web)
    prediction tree [Kronrod-Gotsman 02] (from paper)

Datasets (1)

Datasets (2)

Results: Statistics Summary

- CMST vs. unconstrained MST (lower bound):
  - In most cases: CMST is within 10% of MST
  - On an average: within 17.4%
- Pseudo-CMST vs. flipping & prediction tree (PT):
  - Pseudo-CMST: 8.2—20.41 bits per vertex (b/v)
    Cf. original: 36 b/v
  - Gain over flipping: up to 55.45% (> 32% on an average)
  - Gain over PT: up to 33.17% (> 18% on an average)
  - Also, Pseudo-CMST is very close to original CMST

Conclusions

- Novel geometry compression technique via optimized flipping prediction
- Novel problem formulation & optimization methods
- Geometry oriented, integrating both geometry & connectivity coding
- Large improvements:
  55.45% over flipping; 33.17% over prediction tree

Extension

Tetrahedral meshes (volume data) [Chen-Chiang-Memon-Wu]

Open Problem

Complexity of the CMST problem: NP-complete? Optimal poly.-time algorithm? Approximation algorithm?

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